LETTERS TO THE EDITOR

We encourage readers to express opinions, share insights, and challenge the information that we publish in The Value Examiner. *The advancement of the valuation and forensic accounting professions depends on your ideas and innovations.*

Kasper Responds to Butler's Challenge Regarding Calculation

In January/February 2011, *The Value Examiner* published "A Tale of Two Betas," by Peter J. Butler and Gary Schurman. I presented a critique of that article in "Portfolio Theory and Total Beta," published in the January/February 2012 issue.

The major assertion of my critique was as follows: If one chooses to value a company using the ratio of the standard deviations of a public proxy to the market (Total Beta) in a CAPM formulation of expected return, one must necessarily accept the condition that the private company must be perfectly correlated with the market.

BUTLER'S CHALLENGE

Mr. Butler has challenged me to point out how his calculations of the portfolio standard deviation are incorrect. Before I do, we should note that in his response example, he in fact assumes away the very parameter, λ, which is central to the model's thesis and causes the model's errors. If one assumes λ is 1, "and not worry about lambda," then his calculation of the portfolio variance (and standard deviation) follows the standard formula for the portfolio variance. Unfortunately, his diversification argument depends upon lambda *not* being 1.

The second problem with his challenge is the assumption that the correlation coefficient can be something different than 1 when the company is valued with Total Beta in violation of the proofs in Theorems 2 and 3, and the derivation on pages 29-30 [of the Jan/Feb 2012 issue], for which he had no reply except to claim both are ludicrous. If you start with false premises, the conclusion can never be correct. Nonetheless, for now, I will assume that λ is 1 (which nullifies the diversification argument) and that ρ is not 1, and proceed to examine the variance computation.

Mr. Butler also questions my formula for the variance for a portfolio whose returns are affected by the speculated diversification factor, λ. As proven in my original article, the basic rules of variance and covariance dictate that the traditional portfolio variance becomes

$$[\omega^2\sigma_s^2\lambda^2+(1-\omega)^2\sigma_m^2+2\omega\lambda(1-\omega)\sigma_s\sigma_m\rho]=\sigma_P^2$$

(see derivation in my article, equation 11, page 31, Jan/Feb 2012). Now this formula is not in any text because no one but Messrs. Butler and Schurman have ever proposed such a theory—that a stock's or the portfolio's return should be affected by λ, and hence necessarily, the variance would be affected. This is the formula that must be used in their model to be consistent with the model's claims. If we make λ equal to 1 as Mr. Butler now suggests, this formula collapses to the standard formula for a portfolio variance. Then there is no contradiction between my formula and the traditional formula. Mr. Butler simply assumes away the issue of λ.

ANSWER TO BUTLER'S CHALLENGE

Is Mr. Butler's computation of the portfolio standard deviation value of .2884 incorrect, and more importantly, is it consistent with the premises of the model?

Mr. Butler computes the portfolio variance according to the standard formula, assuming ρ = .50 and the standard deviation as $\sigma_p = .2884$ with investment in the private company, ω = .60. But according to the Butler-Schurman portfolio theory and their equation [B&S 12], $\lambda=(\sigma_p-(1-\omega)\sigma_m)/\omega\sigma_s$ or solving for σ_p $[\omega\sigma_s\lambda+(1-\omega)\sigma_m]=\sigma_p$.

If λ = 1, then according to their model,

$$[\omega\sigma_s(1)+(1-\omega)\sigma_m]=\sigma_p=(.60)(.40)(1)+(1-.60)(.20)=.32$$

Either the model is wrong or Mr. Butler is wrong.

From still another of Mr. Butler's computations, their own model produces inconsistent results. In their original Table 1 (reproduced in part in my article), Mr. Butler gets the same portfolio standard deviation, .2884, with λ equal to .86852. Thus the model produces inconsistent results and is unreliable.

How do you get the same portfolio standard deviation (.2884) with different λ diversification parameters (1 and .86852), or different portfolio standard deviations (.2884 and .32) with the same λ (1)?

Furthermore, according to the premise of the definition of λ, "… the higher the weight, the closer the portfolio gets to a one-asset portfolio where Total Beta (λ = 1) would be the appropriate metric to capture all risk."[1] But a value of λ = 1 should not occur with a weight of ω = 60, but only with weight of ω = 1 (see corollary to Theorem 1). Thus the model is unreliable and inconsistent if the same λ (=1) occurs with different weights (.60 and 1). Furthermore, if ω = 1 as required if λ = 1, then σ_p =.4.

One final observation is needed. The labeled "Correct" values in my Table 2 are close to those of Butler and Schurman *only* because I used the same *incorrect* assumption of ρ = .50 to prove that, even with this assumption, the two calculations for the portfolio return are *not* equal as claimed *and required* by the model, *and* to show further that the return to volatility, S_p, is *not* the market price of risk, .30. Mr. Butler missed the point: the "correct values" are not really correct since ρ is not 1 as required.

SUMMARY

Mr. Butler's own response example shows that the Butler-Schurman portfolio theory is inherently unreliable, unpredictable, and inconsistent. Using the precepts of the model itself, the contradictions are summarized in Exhibit 1.

EXHIBIT 1: INCONSISTENCIES INHERENT IN THE B&S "IMPROVED" PORTFOLIO THEORY

Factor	Result
Different λ (1, .8685)	Same σ_p (.2884)
Same λ (1)	Different σ_p (.2884, .32, .4)
Different weights, ω (.6, 1)	Same λ (1)

1 Peter J. Butler and Gary Schurman, op cit, pg. 24.

Mr. Butler has not shown any errors in my article's derivations of the correct formulas of the portfolio variance and the diversification parameter λ, nor in any of the proofs showing that any company valued by Total Beta requires the assumption that the private company must be perfectly correlated with the market (ρ must be 1) or that the diversification of the buyer is irrelevant.[2]

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Peter J. Butler responds:

In the second paragraph of his letter [above], Mr. Kasper reiterates his major criticism of Total Beta:

> If one chooses to value a company using the ratio of the standard deviations of a public proxy to the market (Total Beta) in a CAPM formulation of expected return, one must necessarily accept the condition that the private company must be perfectly correlated with the market.

I will come back to this incorrect observation below, but most importantly will address his specific criticism of mathematical errors now. Quoting Mr. Kasper's letter:

> Is Mr. Butler's computation of the portfolio standard deviation value of .2884 incorrect, and more importantly, is it consistent with the premises of the model?
>
> If λ = 1, then according to their model,

$$[\omega\sigma_s(1)+(1-\omega)\sigma_m]=\sigma_p=(.60)(.40)(1)+(1-.60)(.20)=.32$$

> Either the model is wrong or Mr. Butler is wrong.
>
> Mr. Kasper forgot about a likely third option: Maybe he

2 In Mr. Butler's current response, footnote 4, he did note that the table value for the rs value with ω = .0 should be .11 and not .11084. However, the correct value was stated in the article as .11 (p. 34). The confusion was my mistake in repeating the value of rs for ω = .3 in ω = .2. The values of rs should be shown on the line above as follows, ω = .2, rs =.12319; ω = .1, rs =.11790; ω = .01, rs =.11084; ω = .0, rs =.1100 due to a simple Excel formula error for those values. The other values are recomputed in a straight forward manner. If my formulas were really incorrect, none of the values of ω between 1 and .3 would be correct either, but Mr. Butler finds no other errors. As I said in the article, the example is not the proof but only an illustration. The proofs remain valid.

is wrong? The computation above is yet another example of where Mr. Kasper loses sight of the big picture and gets bogged down with the math at the expense of common sense.

> From still another of Mr. Butler's computations, their own model produces inconsistent results. In their original Table 1 (reproduced in part in my article), Mr. Butler gets the same portfolio standard deviation, .2884, with λ equal to .86852. Thus the model produces inconsistent results and is unreliable.
>
> How do you get the same portfolio standard deviation (.2884) with different λ diversification parameters (1 and .86852), or different portfolio standard deviations (.2884 and .32) with the same λ (1)?

Answer: You don't unless you mix apples and oranges, as Mr. Kasper does.

In short, there is nothing unreliable here, contrary to Mr. Kasper's views. When I use λ equal to .86852, I correctly get (again) 0.2884, which seems to make sense given the benefits of diversification where λ *goes from 1.0 (with a one-stock portfolio standard deviation of 0.40, which is not equal to* ***0.32****,* as calculated above by Mr. Kapser) to 0.86852 (with a portfolio standard deviation of 0.2884). Please see Table 1 in our original article way back in the January/February 2011 issue of *The Value Examiner* for more details.

Since Mr. Kasper seemingly agrees that λ should only equal 1.0 when the investor has a single stock portfolio, it is rather troubling that he uses a λ of 1.0 with a stock weighting of 0.6 (i.e., not a single stock portfolio) to try to prove his point that we calculated portfolio standard deviation incorrectly, counter to common sense and as shown below. (I use the symbol σ, consistent with our first article on this topic, in lieu of Mr. Kasper's symbol, s).

$$\lambda = (\sigma_p - (1- \omega)\sigma_m)/\omega\sigma_s$$

If ω = 1.0, meaning the investor only owns a private company,

$$\text{then } \lambda = (\sigma_p - (1- 1)\sigma_m)/1\sigma_s = \sigma_p/\sigma_s$$

and since σ_s is σ_p, λ must be 1.0 only in this scenario—not when ω = 0.6 or any other weighting less than 100 percent, making his calculation above equal to 0.32 pointless and confusing.

In my first response, when I say, "Let's not worry about lamda," it doesn't mean that I assume it is 1.0 as Mr. Kasper would like you to believe; it means that I am using a completely different formula to calculate portfolio standard deviation, much as you can calculate Total Beta by using relative standard deviations, σ_s/σ_m, or using Beta/ρ_{sm}. If I calculate Total Beta using standard deviations, then I do not have to "worry about Beta." It does not mean that Beta is 1.0. It just means I did not use it in my formula. Thus, I never make the assumption that lamda is 1.0 unless it is a one-stock portfolio, contrary to Mr. Kasper's odd and misguided calculations. In any event, up and down our original Table 1, I get the same answers for the portfolio standard deviations using traditional financial theory or our formula using lamdas since I do not mix apples with oranges.

So, I am going to stop here and concentrate on the bigger picture for the benefit of the valuation community.

Mr. Kasper believes that a private company has to be perfectly correlated with the market to use Total Beta because he incorrectly treats every private company like it was a publicly traded company. I believe the following to be his train of thought.

Total Beta is set forth as β/ρ_{sm} where β is the company's Beta and ρ_{sm} is the correlation coefficient between the stock and the market. In Mr. Kasper's (public stock) world, he cannot accept anything else besides Beta to be used as a multiplier on the equity risk premium ($R_m - r_f$). So, he claims ρ_{sm} must be equal 1.0 to use Total Beta. You see if ρ_{sm} is equal to 1.0, everything is perfect in Mr. Kasper's world because then Total Beta ($\beta/\rho_{sm} = \beta/1.0 = \beta$) equals Beta. Now and only now, when Beta equals Total Beta, will Mr. Kasper use Total Beta. But he is missing the big picture, as well as the fact that Total Beta proponents readily admit we are violating the CAPM by replacing Beta with Total Beta. Yet Mr. Kasper always wants to squeeze us back into the CAPM framework to justify his criticisms and support his (questionable) math, and we do not have to go there for private company valuation since we price for company specific risk (CSR).

What if we assume a different perspective? What if we simply make the following assumption—a reasonable assumption by all Total Beta proponents—as opposed to rather subjectively guessing at an appropriate CSRP?

Total Beta proponents merely assume (remember, it is a model) that CSR is priced according to the market price of risk. Another way to look at this assumption is the following, which has been up front and center since the "invention" of Total Beta.

Hypothetically, an investor can choose to invest in the market (S&P 500) and be completely (or at least practically) diversified, or hold just one stock and be completely undiversified. For this investor to be ambivalent between the choice of investments, the following formula must hold, where "s" stands for stock, "m" for market, σ_s = standard deviation of the stock, σ_m = the standard deviation of the market, R_m represents the market rate of return, R_f represents the risk-free rate, and, of course, TCOE stands for the total cost of equity of the single stock portfolio:

$$(R_m - R_f)/\sigma_m = (TCOE - R_f)/\sigma_s$$

which results in:

$$TCOE = R_f + (\sigma_s/\sigma_m)(R_m - R_f)$$

which should look more familiar as:

$$TCOE = R_f + (\text{Total Beta})(\text{equity risk premium})$$

Hence, Professor Damodaran created a new *model* for private-company valuation with really just one additional, simple assumption: that CSR (and/or total risk: $(TCOE - R_f)/\sigma_s$) is priced like the market price of risk $(R_m - R_f)/\sigma_m$. Since CSR is not priced on an ex-ante basis for publicly traded stocks, no one can refute this ex-ante assumption for private-company valuation as being incorrect. Moreover, if someone has a better idea than this reasonable assumption, then please disclose it to the business valuation community.

In the meantime, I will continue to use Total Beta (and/or private-company Beta,[3] which accounts for partial diversification), as I have only grown more confident in their use through the years—from both a practical and theoretical perspective—as opposed to completely guessing at a CSRP.

After all, when appraisers add a completely subjective CSRP to their discount rate, how have they priced the CSRP? At least Total Beta proponents know. VE

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3 Total Beta and private-company Beta are reasonably close in magnitude if reasonable assumptions are made regarding the weighting of the business in a portfolio.